Discussion on the two-phase model of nonuniformly fluidized beds according to Pyle and Harrison

[a]O. MIKULA and [b]I. HAVALDA

[a]*Department of Chemical Engineering, Refinery and Petrochemical Company, Slovnaft n.e., 823 00 Bratislava*

[b]*Department of Chemical Engineering, Slovak Technical University, 880 37 Bratislava*

Received 29 January 1971

Accepted for publication 5 May 1973

The values of relative expansions found experimentally in the systems water—particles and air—particles were compared with the equations for theoretical relative expansion derived by *Pyle* and *Harrison* [8] on the basis of two-phase model of nonuniformly fluidized beds. A disagreement between theoretical equation and visual observation appears.

During the last 20 years a number of researchers studied fluidized beds; so far they have not succeeded in developing a satisfactory hydrodynamic model. Recently the so-called two-phase model of nonuniformly fluidized beds attracted attention. It is based on visual observation as well as on the measurements of physical characteristics of beds by capacitance technique and by optical, X-rays, and radioisotopic methods. These measurements were performed by several authors, namely *Morse* and *Ballou* [1], *Dotson* [2], *Toomey* and *Johnstone* [3], *Romero* and *Johanson* [4], *Lanneau* [5], *Baumgarden* and *Pigford* [6], *Harrison*, *Davidson*, and *de Kock* [7] *et al.*

The concept of a nonuniformly fluidized bed according to two-phase model may be expressed as follows. A nonuniformly fluidized bed consists of two regions:

a) a dense suspension which is often denoted improperly as a continuous phase or dense phase;

b) a dilute suspension which is often denoted improperly as a dilute phase.

The flow-rate of fluid through the dense suspension equals the flow-rate through the bed at the incipient fluidizing velocity Q_i. The remainder of fluid Q_b exceeding the incipient fluidizing velocity volume flow passes through the bed discontinuously as bubbles and produces a dilute suspension. The total flow-rate of fluid Q may then be expressed by the relationship

$$Q = Q_b + Q_i. \qquad (1)$$

By dividing this equation by the cross section of column S we obtain

$$w = w_b + w_i, \qquad (2)$$

where w is the superficial velocity of fluid in the column, w_b is the velocity of fluid (referred to the cross section of column) flowing in bubbles, and w_i is the incipient fluidizing velocity.

The equation (*1*) has to be considered an unverified hypothesis which defines Q_b. This definition of Q_b from which the definition of w_b according to the equation (*2*) ensues is a generally accepted hypothesis of the two-phase model.

However, the solution of individual problems requires the introduction of further independent quantities and their definitions which would enable to get different modifications of the two-phase model as well as various adequate relationships among the quantities. For instance, the volume of bubbles in bed is of importance for expressing the height L of nonuniformly fluidized bed. This volume may be determined if the rising velocity of bubbles in bed u_b is known. On the basis of incomplete experimental knowledge *Pyle* and *Harrison* [8] put

$$u_b = u_{bi} + K\, w_b, \tag{3}$$

where the dimensionless coefficient K assumes the values 1.00 and 1.20 for a two- or three-dimensional bed, respectively. The symbol u_{bi} stands for the rising velocity of separate bubble through the bed in the proximity of the incipient fluidization which depends merely on the acceleration due to gravity g and the inner diameter of column D; for two- or three-dimensional bed the velocity is given by

$$u_{bi} = 0.23(g\, D)^{0.5}, \tag{4}$$

or

$$u_{bi} = 0.35(g\, D)^{0.5}, \tag{5}$$

respectively.

Assuming that all bubbles are of equal size (and move with the same velocity), the subsequent equation ensues from equations (*1*—*5*) for expansion

$$\frac{L_i}{L} = 1 - \frac{w - w_i}{K\, w - w_i - u_{bi}}\,. \tag{6}$$

Since according to the authors [8] the bubbles are of different size and move with different rising velocity while the equations (*3*—*5*) define the maximum velocity, the quantity L in equation (*6*) represents the minimum value of the bed height.

On the basis of graphical comparison of the theoretical expansions according to equation (*6*) with the values presented by *Pyle* and *Harrison* [8] it is not possible to judge objectively the accuracy of the two-phase model because the authors give no data concerning the method of measurement, the kind of fluid used, the parameters of particles, *etc.*

The comparison of our experimental values found for the systems water—particles and air—particles with equation (*6*) gives a new approach to the accuracy of the assumptions of *Pyle* and *Harrison* [8].

Experimental

The measurements were carried out with systems water—particles (glass column 0.01148 m in diameter; height 1.01 m; brass grid; screened glass Ballotini; fractions ranging from 1.260 to 21.45 $\times$ 10^{-4} m in diameter) and air—particles (perspex glass column of inner diameter 0.1101 m; height 1.55 m; screened glass Ballotini; fractions from 1.265 to 4.424 $\times$ 10^{-4} m). The characteristics of the particles used are given in Table 1.

Table 1

Denotation and characteristics of the samples of the used glass particles

Sample	Effective diameter of particles $d \cdot 10^4$ [m]	Density of particles ϱ_p [kg/m³]	Archimedes number Ar	Fluidization liquid
A_1	1.260	2778.0	30	water
A_2	2.130	2726.0	168	water
A_3	4.650	2744.0	1 613	water
A_4	6.110	2729.9	2 758	water
A_5	11.48	2737.0	20 810	water
A_6	21.45	2572.0	135 000	water
B_1	1.265	2615.3	197	air
B_2	1.528	2624.9	350	air
B_3	2.078	2684.3	908	air
B_4	4.424	2697.9	8 955	air

The nonuniformly fluidized beds manifest themselves [9] by the fluctuation of the expanded bed height between L_{max} and L_{min}. In a sufficiently long time interval of observation T (5 minutes) these boundary values are fairly reproducible. The heights L_{max} and L_{min} were measured visually by means of a sliding paper strip. From the measured values the average height of expanded bed L_a was calculated as an arithmetic mean

$$L_a = \frac{L_{max} + L_{min}}{2}. \tag{7}$$

In systems fluidized by air the beginning of the fluctuation of the bed height was observed immediately above the incipient fluidizing velocity.

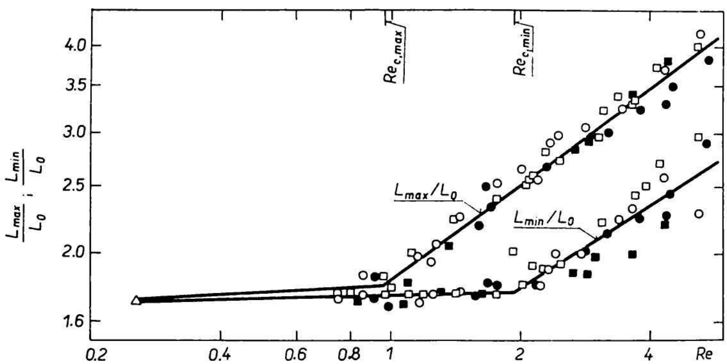

Fig. 1. Experimental dependence of the relative maximum (L_{max}/L_0) and minimum (L_{min}/L_0) height of fluidized bed on Reynolds number (Re) for the system air—particles at various sample weights m of the particles B_2.
○ m = 1.50 kg; □ m = 2.00 kg; ● m = 2.50 kg; ■ m = 4.00 kg; △ incipient fluidizing velocity.
— graphical representation of equations (*16*), (*17*), (*18*), and (*19*); Table 2.

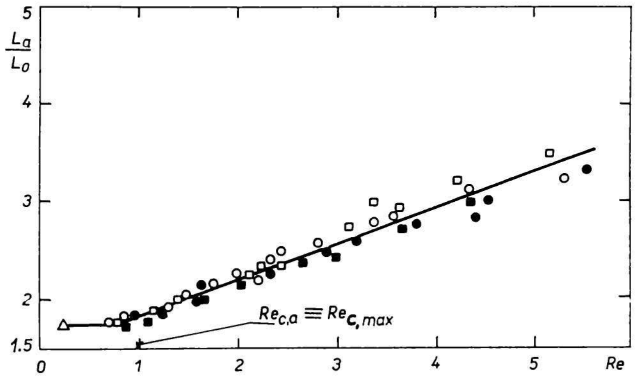

Fig. 2. Experimental dependence of the relative mean height (L_a/L_0) of fluidized bed on Reynolds number (Re) for the system air—particles at various sample weights m of the particles B_2.
○ $m = 1.50$ kg; $m = 2.00$ kg; ● $m = 2.50$ kg; ■ $m = 4.00$ kg; incipient fluidizing velocity.
—— graphical representation of equations (*20*) and (*21*); Table 2.

Figs. 1 and 2 show the results of measurements obtained with the system air—particles for a sample of particles B_2 as relationships

$$\log\,[L_{max}/L_0] = \mathrm{f}\,(\log Re),$$

$$\log\,[L_{min}/L_0] = \mathrm{f}\,(\log Re)$$

or

$$L_a/L_0 = \mathrm{f}\,(Re),$$

where L_0 is the height of compact bed of particles.

Similar results were obtained also with the samples of particles B_1, B_3, and B_4. It is worth noticing that the values L_{max}/L_0, L_{min}/L_0, and L_a/L_0 are practically independent of the weight of particles m or L_0. From the distribution of points in Figs. 1 and 2 it may be concluded that these relationships change abruptly which may be expressed by two power equations of the type

$$\log y = \log a_2 + a_1 \log x \tag{8}$$

for the relationships

$$\log\,[L_{max}/L_0] = \mathrm{f}\,(\log Re)$$

and

$$\log\,[L_{min}/L_0] = \mathrm{f}\,(\log Re)$$

or by two linear equations of the type

$$y = b_2 + b_1 \tag{9}$$

for the relationship

$$L_a/L_0 = \mathrm{f}\,(Re).$$

The values of the parameters a_1, a_2, b_1, and b_2 for individual series of measurements which were calculated as arithmetic means or by the method of least squares are given in

Table 2

Results of mathematical evaluation of the measurements in the system air—particles

Sample			B_1	B_2	B_3	B_4
$Ar =$			197	350	908	8955
$L_{min}/L_0 = f(Re)$	$Re_i \leq Re \leq$ $\leq Re_{c,min}$	$L_{min}/L_0 =$	$1.76\ Re^{0.0205}$	$1.73\ Re^{0.0280}$	$1.71\ Re^{0.0397}$	$1.58\ Re^{0.0493}$
		Equation	*(10)*	*(16)*	*(22)*	*(28)*
	$Re_{c,min} =$		1.56	1.94	4.11	9.92
	$Re >$ $> Re_{c,min}$	$L_{min}/L_0 =$	$1.48\ Re^{0.411}$	$1.36\ Re^{0.397}$	$0.866\ Re^{0.522}$	$0.691\ Re^{0.409}$
		Equation	*(11)*	*(17)*	*(23)*	*(29)*
$L_{max}/L_0 = f(Re)$	$Re_i \leq$ $\leq Re \leq$ $\leq Re_{c,max}$	$L_{max}/L_0 =$	$1.83\ Re^{0.0373}$	$1.82\ Re^{0.0597}$	$1.76\ Re^{0.0773}$	$1.56\ Re^{0.0600}$
		Equation	*(12)*	*(18)*	*(24)*	*(30)*
	$Re_{c,max} =$		0.731	0.975	1.52	6.42
	$Re >$ $> Re_{c,max}$	$L_{max}/L_0 =$	$2.08\ Re^{0.450}$	$1.84\ Re^{0.456}$	$1.50\ Re^{0.457}$	$0.506\ Re^{0.667}$
		Equation	*(13)*	*(19)*	*(25)*	*(31)*
$L_a/L_0 = f(Re)$	$Re_i \leq$ $\leq Re \leq$ $\leq Re_{c,a}$	$L_a/L_0 =$	$1.71 + 0.0975\ Re$	$1.70 + 0.0773\ Re$	$1.67 + 0.0810\ Re$	$1.72 + 0{,}00054\ Re$
		Equation	*(14)*	*(20)*	*(26)*	*(32)*
	$Re_{c,a} = Re_{c,max} =$		0.731	0.974	1.52	6.42
	$Re >$ $> Re_{c,a}$	$L_a/L_0 =$	$1.45 + 0.469\ Re$	$1.46 + 0.362\ Re$	$1.44 + 0.228\ Re$	$1.07 + 0.101\ Re$
		Equation	*(15)*	*(21)*	*(27)*	*(33)*

equations (*10—33*) in Table 2. The relationships according to the equations in Table 2 are drawn as full lines in Figs. 1 and 2 and correspond to the sample of particles B_2.

The values of Reynolds numbers at which the break appears according to Figs. 1 and 2 are denoted $Re_{c,max}$, $Re_{c,min}$, $Re_{c,a}$ and increase with the value of the number Ar. In our experiments $Ar \in \langle 197; 8955 \rangle$; therefore no profounder quantitative conclusions concerning this relationship may be drawn.

In systems fluidized by water the beginning of the fluctuation of the bed height was always observed visually in the so-called transition region of flow which is limited by the interval $Re_{c_1} < Re < Re_{c_2}$, Re_{c_1} and Re_{c_2} being the first and the second critical Reynolds number according to *Beňa et al.* [10].

Discussion

By inserting from (*4*) or (*5*) into (*6*) and introducing a dimensionless complex Z

$$Z = \frac{w - w_i}{(g\ D)^{0.5}} \tag{34}$$

equation (*6*) may be transformed either into the form

$$\frac{L_i}{L} = 1 - \frac{Z}{1.00\ Z + 0.23} \tag{35}$$

valid for the two-dimensional bed or into the form

$$\frac{L_i}{L} = 1 - \frac{Z}{1.20\ Z + 0.35} \tag{36}$$

valid for the three-dimensional bed.

The experimental values of L_i/L_a obtained with fluidized beds in the systems water—particles and air—particles are compared according to equations (*35*) and (*36*) in Fig. 3. (The experimental values of L_i/L_a or L_i/L_{max} obtained for the system air—particles were smoothed according to equations in Table 2.)

According to *Pyle* and *Harrison* [8] the theoretical curve $L_i/L = f(Z)$ at a certain value of Z represents the mean or at least the minimum height of bed, L, or the maximum value of the relative expansion of bed, L_i/L. As obvious from Fig. 3, this theoretical prediction disagrees with the facts observed. For the fluidized systems water—particles the values of L_i/L_a are much lower over the whole range of the Z values or the values L_a are much higher than theoretically assumed. The disagreement between the theoretical prediction and the experimental results decreases with increasing value of Archimedes number Ar.

In the fluidized systems air—particles where the fluctuation of the bed height was observed visually just above the incipient fluidizing velocity, the agreement between the values of L_i/L postulated by theory and those obtained experimentally was better than in the case of water-fluidized systems; notwithstanding it was not satisfactory. However, it must be emphasized that the assumption of *Pyle* and *Harrison* [8] according to which the value of L_i/L calculated from equations (*35*) and (*36*) ought to be at least maximum gets entirely refuted. While in systems fluidized by water the experimental value of L_i/L_a approaches the theoretical value of L_i/L with increasing value of Ar (Z = const), in the air-fluidized systems the experimental value of L_i/L_a falls

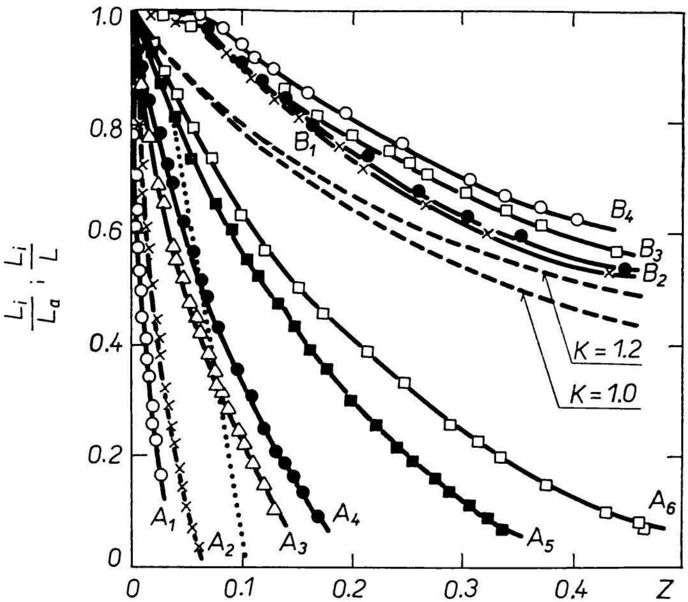

Fig. 3. Comparison of the measured values of L_i/L_a in the systems water—particles and air—particles with the theoretical values of L_i/L according to *Pyle* and *Harrison* [8] in dependence on the dimensionless complex Z defined by equation (*34*).
$A_1 - A_6$ and $B_1 - B_4$ — denotation of the samples of particles;
— — — — — graphical representation of equations (*35*) and (*36*);
experimentally observed boundary between uniformly (on the left side of line) and nonuniformly (on the right side of line) fluidized beds in the system water—particles.

back from the theoretical value with increasing value of Ar. It is obvious that the disagreement in the systems air—particles would have been even greater if we compared the maximum experimental values of L_i/L_{min} (instead of mean experimental values of L_i/L_a as seen in Fig. 3) with expected theoretical values because the curves $L_i/L_{min} = f(Z)$ would have been more distant from the theoretical curves $L_i/L = f(Z)$. In the systems water—particles the situation would have not changed substantially because L_{min} was only little different from L_a.

According to the existing knowledge and our considerations [9] concerning two kinds of disturbance forces which cause the nonuniformity of fluidized beds the two-phase model of Pyle and Harrison may be evaluated as follows.

1. In the nonuniformly fluidized systems fluid—particles equations (*1*) and (*2*) are not valid; that means that the basic concept of the two-phase model is not correct. Equations (*3*—*5*) do not express the physical reality and equation (*6*) cannot be in force.

The apparent parametric dependence of $L_i/L = f(Z)$ on Ar which seems to exist according to Fig. 3 may be explained on the basis of the statement of several authors that for the ratio $u_t/w_t = \alpha$ the value of α decreases with increasing value of Ar in a certain fairly wide interval of Archimedes numbers (which includes the series of our measurements). Hence

$$u_t - w_i = w_i(\alpha - 1), \tag{37}$$

or

$$Z_{max} = \frac{w_i(\alpha - 1)}{(g\ D)^{0.5}}, \tag{38}$$

where α may depend on Archimedes number. In addition, for particles of certain shape the quantity depends on ϱ_f, μ, d, and Ar and for this reason for D = const Z_{max} depends on ϱ_f, μ, d, and Ar. Generally, Z can change in the interval $Z \in \langle 0; Z_{max}\rangle$ but irrespective of the values Z_{max}, L_i/L_a assumes the values from the interval $\langle 0; 1\rangle$. Therefore for higher values Z_{max} at D = const a less steep slope of the function $L_i/L_a =$ $=$ f(Z) has to be expected for the system water—particles in conformity with Fig. 3.

In the system air—particles (at low pressures) different course [9] has to be expected because the interval of the values L_i/L_a is differently limited, *i.e.* $w \to u_t$ does not hold provided $L_i/L_a \to 0$.

2. Let us admit that for the systems gas—particles (at low pressures) the basic concept of the nonuniformly fluidized bed according to two-phase model is correct, *i.e.* equations (*1*) and (*2*) are valid. (It is difficult to provide a direct experimental evidence of the accuracy of these equations; this hypothesis has been neither confirmed nor refuted up to now.) Then it follows from Fig. 3 that equations (*3—5*) are not correct and the problem of a successful two-phase model appears as a problem of a correct definition of the rising velocity of bubbles u_b. In this sense, attempts to refine this model should be made.

Symbols

Ar	$= [g\ d^3(\varrho_p - \varrho_f)\ \varrho_f]\ \mu^{-2}$ — Archimedes number
a_1, a_2	parameters in equation (*8*)
b_1, b_2	parameters in equation (*9*)
d	diameter or effective diameter of particle
D	inner diameter of column
g	acceleration due to gravity
K	dimensionless coefficient in equation (*3*)
L	bed height
L_{max}, L_{min}	maximum and minimum height of the nonuniformly fluidized bed observed after a sufficiently long time interval under constant conditions
L_0	$= 4m/\pi\ D^2\ \varrho_p$ — height of compact bed of particles
L_i	bed height at incipient fluidization
L_a	average bed height defined by equation (*7*)
m	mass of the sample of particles
Q	total flow-rate of fluid through the bed
Q_b	flow-rate of fluid through the bed in bubble phase
Q_i	value of Q at incipient fluidization
Re	$= wd\ \varrho_f\ \mu^{-1}$ — Reynolds number
Re_{c1}	$= 0.0157\ Ar^{0.698} + 0.400$ — the first critical Reynolds number
Re_{c2}	$= 0.192\ Ar^{0.548} - 1.00$ — the second critical Reynolds number
$Re_{c,max}$	Reynolds number characterizing the break of the curve $\log\ [L_{max}/L_0] =$ $=$ f ($\log Re$)

$Re_{c,min}$	Reynolds number characterizing the break of the curve $\log [L_{min}/L_0] = f(\log Re)$
$Re_{c,a}$	Reynolds number characterizing the break of the curve $L_a/L_0 = f(Re)$
Re_i	$= w_i d \varrho_f \mu^{-1}$ — Reynolds number at incipient fluidization
S	cross section of column
u_{bi}	rising velocity of a separate bubble in the vicinity of the incipient fluidization
u_t	terminal velocity of separate particle in viscous medium
w	superficial fluid velocity in the column
w_b	superficial fluid velocity in bubble phase
w_i	incipient fluidizing velocity
Z	dimensionless complex defined by equation (*34*)
Z_{max}	dimensionless complex defined by equation (*38*)
α	ratio u_t/w_i
μ	dynamic viscosity of fluid
ϱ_f	density of fluid
ϱ_p	density of particles

References

1. Morse, R. D. and Ballou, C. O., *Chem. Eng. Progr.* ***41***, 199 (1951).
2. Dotson, J. M., *A. I. Ch. E. Journal* ***5***, 169 (1959).
3. Toomey, R. D. and Johnstone, H. F., *Chem. Eng. Progr.* ***48***, 220 (1952).
4. Romero, J. B. and Johanson, L. N., *Chem. Eng. Progr., Symp. Ser.* ***58***, 28 (1962).
5. Lanneau, K. P., *Trans. Inst. Chem. Eng.* ***38***, 125 (1960).
6. Baumgarden, P. K. and Pigford, R. L., *A. I. Ch. E. Journal* ***6***, 115 (1960).
7. Harrison, D., Davidson, J. F., and de Kock, J. W., *Trans. Inst. Chem. Eng.* ***39***, 202 (1961).
8. Pyle, D. L. and Harrison, D., *Chem. Eng. Sci.* ***22***, 1199 (1967).
9. Mikula, O., Beňa, J., and Havalda, L., *Collect. Czech. Chem. Commun.* ***37***, 2343 (1972).
10. Beňa, J., Ilavský, J., Kossaczký, E., and Neužil, L., *Collect. Czech. Chem. Commun.* ***28***, 293 (1963).

Translated by R. Domanský